



INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

SENIOR PAPER: YEARS 11,12

Tournament 40, Northern Autumn 2018 (O Level)

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Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Is it possible to place a line segment XY inside a regular pentagon $A_1A_2A_3A_4A_5$ so that all five angles $\angle XA_iY$ ($i = 1, \dots, 5$) are equal? (3 points)
2. Determine all positive integers n such that the numbers $1, 2, \dots, 2n$ can be divided into pairs so that the product of sums of the numbers in each pair is a perfect square. (4 points)
3. In parallelogram $ABCD$, $\angle A$ is acute. A point N is chosen on the side AB so that $CN = AB$. Suppose that the line AD is tangent to the circumcircle of triangle CBN . Prove that D is the point of tangency. (5 points)
4. A nine-digit integer is called *beautiful* if all of its digits are different. Prove that there exist at least 1000 beautiful numbers, each of which is divisible by 37. (5 points)
5. Petya is placing 500 kings on a 100×50 chess board so that the kings don't attack one another. Vasya is placing 500 kings on white squares of a 100×100 chess board so that the kings don't attack one another. Who has more ways to place the kings? (5 points)